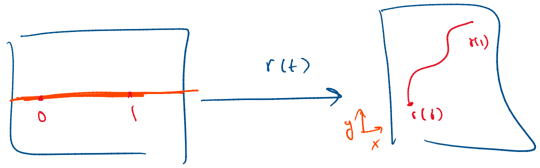
Math 53, Discussion 105

Monday, April 20, 2020 9:26 AM

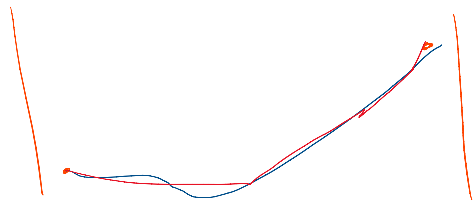
16.5 (divergence and curl)

16.6 (parametric surfaces and their areas)

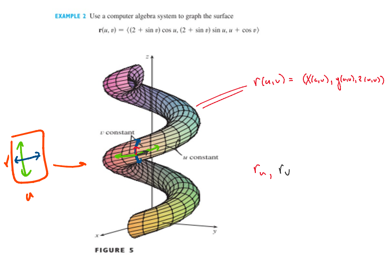


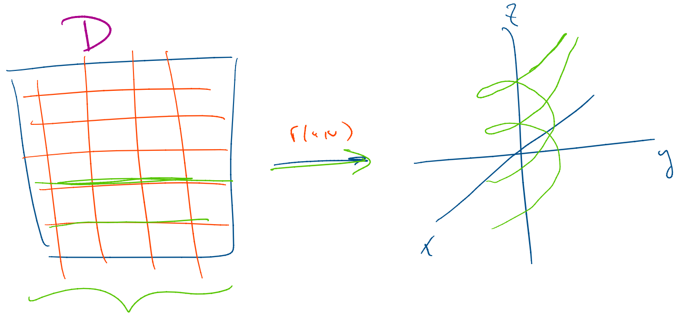
$\int f(r(t))\,|r'(t)|\,dt$

$\left.\begin{array}{l}\langle \cos(2t), \sin(2t)\rangle \\ \langle \cos(t), \sin(t)\rangle\end{array}\right\}$ not unique to parametrn.

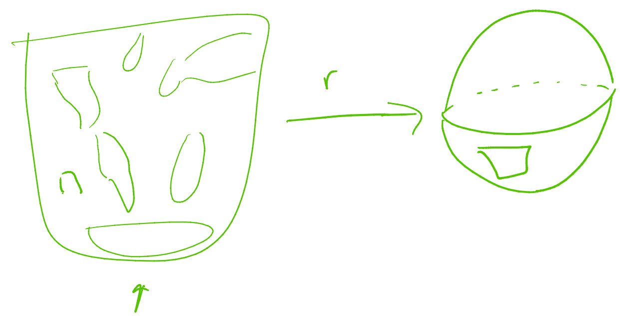


Def: a parametric surface is a function $r: R^2 \to R^3, r(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$





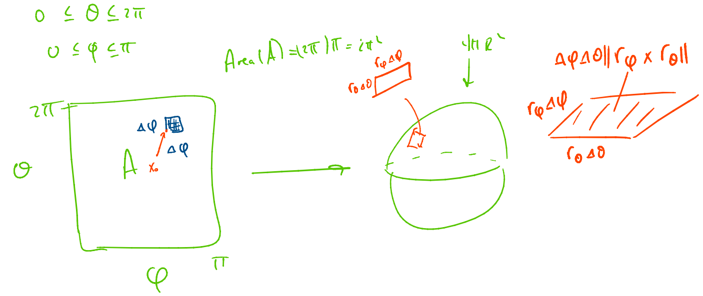
By computing $\frac{\partial r}{\partial u}$ and $\frac{\partial r}{\partial v}$ we can get formulae for the tangent plane and surface area.



$r(u,v) = \langle R\cos\theta\sin\varphi, R\sin\theta\sin\varphi, R\cos\varphi\rangle$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \pi$



Param. surface

$r(u_0, v_0)$

$r_u(r_0) \times r_v(r_0)$

Tangent plane of r at r_0:

Surface area of r

$\iint dA$

Surface area of r

$$\iint_D dA$$

$$dA = \| r_u \times r_v \| \, du \, dv$$

Exercises:

1. Compute $\int_C y^3 dx - x^3 dy$ where C is the circle centered at the origin of radius 2
2. Determine whether $F = \langle e^z, y, xe^z \rangle$ is conservative, if it is, find its potential
3. Identify the surface with the given vector equation $\langle 2\sin u, 3\cos u, v\rangle, 0 \le v \le 2$

$-3x^2 - 3y^2 \quad -3\iint x^2+y^2 = -3\iint r^3 \, dr\, d\theta$

$= \boxed{-6\pi \frac{2^4}{4}} = -24\pi$

$f = xe^z + \frac{y^2}{2}$

$$\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} e^z \\ y \\ xe^z \end{pmatrix} = \begin{pmatrix} 0 \\ e^z - e^z \\ 0 \end{pmatrix} = 0$$

13–18 Match the equations with the graphs labeled I–VI and give reasons for your answers. Determine which families of grid curves have u constant and which have v constant.

13. $\mathbf{r}(u, v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + v\,\mathbf{k}$

14. $\mathbf{r}(u, v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + \sin u\,\mathbf{k}, \quad -\pi \le u \le \pi$

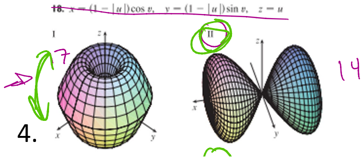
15. $\mathbf{r}(u, v) = \sin v\,\mathbf{i} + \cos u \sin 2v\,\mathbf{j} + \sin u \sin 2v\,\mathbf{k}$

16. ~~$x = (1-u)(3+\cos v)\cos 4\pi u$, $y = (1-u)(3+\cos v)\sin 4\pi u$, $z = 3u + (1-u)\sin v$~~

17. $x = \cos^3 u \cos^3 v, \quad y = \sin^3 u \cos^3 v, \quad z = \sin^3 v$

18. ~~$x = (1-|u|)\cos v, \quad y = (1-|u|)\sin v, \quad z = u$~~

$\| \langle \cos^3 \cdot \cos^3, \sin \quad , \quad \rangle \|$

4. 

4.